MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

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Year 1

1.1 Comparison of quantities and measures

Explore the relationship between numbers and introduce children to the important concept of equivalence; focus on the correct use of comparative language, as well as use of mathematical symbols (<, = and >).

- **Teaching point 1:** Items can be compared according to attributes such as length (or height or breadth), area, volume/capacity or weight/mass.
- **Teaching point 2:** When comparing two sets of objects, one set can contain more objects than the other and one set can contain fewer objects than the other, or both sets can contain the same number of objects.
- **Teaching point 3:** The symbols <, > and = can be used to express the relative number of objects in two sets, or the relative size of two numbers.

1.2 Introducing 'whole' and 'parts': part–part–whole

Introduce children to the concept of partitioning, which underpins many of the subsequent segments, and build towards use of the part–part–whole model.

- **Teaching point 1:** A 'whole' can be represented by one object; if some of the whole object is missing, it is not the 'whole'.
- **Teaching point 2:** A whole object can be split into two or more parts in many different ways. The parts might look different; each part will be smaller than the whole, and the parts can be combined to make the whole.
- **Teaching point 3:** A 'whole' can be represented by a group of discrete objects. If some of the objects in the group are missing, it is not the whole group – it is part of the whole group.
- **Teaching point 4:** A whole group of objects can be composed of two or more parts and this can be represented using a part–part–whole 'cherry' diagram. The group can be split in many different ways. The parts might look different; each part will be smaller than the whole group and the parts can be combined to make the whole group.

1.3 Composition of numbers: 0–5

Apply the partitioning structure to the numbers to five, and introduce children to new concepts such as subitising, ordinality and the bar model.

- **Teaching point 1:** Numbers can represent how many objects there are in a set; for small sets we can recognise the number of objects (subitise) instead of counting them.
- **Teaching point 2:** Ordinal numbers indicate a single item or event, rather than a quantity.
- **Teaching point 3:** Each of the numbers one to five can be partitioned in different ways.
- **Teaching point 4:** Each of the numbers one to five can be partitioned in a systematic way.
- **Teaching point 5:** Each of the numbers one to five can be partitioned into two parts; if we know one part, we can find the other part.
- **Teaching point 6:** The number before a given number is one less; the number after a given number is one more.
- **Teaching point 7:** Partitioning can be represented using the bar model.

1.4 Composition of numbers: 6–10

Extend the partitioning structure to the numbers six to ten, explore the five-and-a-bit structure of the numbers, and introduce children to the concept of odd and even numbers.

- **Teaching point 1:** The numbers six to nine are composed of 'five and a bit'. Ten is composed of five and five.
- **Teaching point 2:** Six, seven, eight and nine lie between five and ten on a number line.
- **Teaching point 3:** Numbers that can be made out of groups of two are even numbers; numbers that can't be made out of groups of two are odd numbers. Even numbers can be partitioned into two odd parts or two even parts; odd numbers can be partitioned into one odd part and one even part.
- **Teaching point 4:** Each of the numbers six to ten can be partitioned in different ways. The numbers six to ten can be partitioned in a systematic way.
- **Teaching point 5:** Each of the numbers six to ten can be partitioned into two parts; if we know one part, we can find the other part.

1.5 Additive structures: introduction to aggregation and partitioning

Progress to the use of abstract notation (+, – and =) as a way of representing the part–part–whole structure.

- **Teaching point 1:** Combining two or more parts to make a whole is called aggregation; the addition symbol, +, can be used to represent aggregation.
- **Teaching point 2:** The equals symbol, =, can be used to show equivalence between the whole and the sum of the parts.
- **Teaching point 3:** Each addend represents a part, and these are combined to form the whole/sum; we can find the value of the whole by adding the parts. We can represent problems with missing parts using an addition equation with a missing addend.
- **Teaching point 4:** Breaking a whole down into two or more parts is called partitioning; the subtraction symbol, –, can be used to represent partitioning.

1.6 Additive structures: introduction to augmentation and reduction

Introduce children to addition as augmentation, and subtraction as reduction (take away), using a *'first…, then…, now…'* story representation and abstract notation (+, – and =); explore the inverse nature of the two operations.

- **Teaching point 1:** an addition context described by a *'first…, then…, now…'* story is an example of augmentation. We can link the story to a numerical representation – each number represents something in the story.
- **Teaching point 2:** a subtraction context described by a *'first…, then…, now…'* story is an example of reduction. We can link the story to a numerical representation – each number represents something in the story.
- **Teaching point 3:** given any two parts of the story we can work out the third part; given any two numbers in the equation we can find the third one.
- **Teaching point 4:** addition and subtraction are inverse operations.

1.7 Addition and subtraction: strategies within 10

Equip children with a range of useful strategies for addition within ten, including adding and subtracting zero and one, commutativity, adding and subtracting two to/from odd and even numbers, and doubling and halving.

- **Teaching point 1:** Addition is commutative: when the order of the addends is changed, the sum remains the same.
- **Teaching point 2:** Ten can be partitioned into pairs of numbers that sum to ten. Recall of these pairs of numbers supports calculation.
- **Teaching point 3:** Adding one gives one more; subtracting one gives one less.
- **Teaching point 4:** Consecutive numbers have a difference of one; we can use this to solve subtraction equations where the subtrahend is one less than the minuend.
- **Teaching point 5:** Adding two to an odd number gives the next odd number; adding two to an even number gives the next even number. Subtracting two from an odd number gives the previous odd number; subtracting two from an even number gives the previous even number.
- **Teaching point 6:** Consecutive odd / consecutive even numbers have a difference of two; we can use this to solve subtraction equations where the subtrahend is two less than the minuend.
- **Teaching point 7:** When zero is added to a number, the number remains unchanged; when zero is subtracted from a number, the number remains unchanged.
- **Teaching point 8:** Subtracting a number from itself gives a difference of zero.
- **Teaching point 9:** Doubling a whole number always gives an even number and can be used to add two equal addends; halving is the inverse of doubling and can be used to subtract a number from its double. Memorised doubles/halves can be used to calculate near-doubles/halves.
- **Teaching point 10:** Addition and subtraction facts for the pairs five and three, and six and three, can be related to known facts and strategies.

1.8 Composition of numbers: multiples of 10 up to 100

Explore multiples of ten, including counting in tens to 100; apply number facts within ten to addition and subtraction for multiples of ten.

- **Teaching point 1:** One ten is equivalent to ten ones.
- **Teaching point 2:** Multiples of ten can be represented using their names or using numerals. We can count in multiples of ten.
- **Teaching point 3:** Knowledge of the 0–10 number line can be used to estimate the position of multiples of ten on a 0–100 number line.
- **Teaching point 4:** Adding ten to a multiple of ten gives the next multiple of ten; subtracting ten from a multiple of ten gives the previous multiple of ten.
- **Teaching point 5:** Known facts for the numbers *within* ten can be used to add and subtract in multiples of ten by unitising.

1.9 Composition of numbers: 20–100

Build on multiples of ten, by introducing non-zero values in the ones place; apply the partitioning structure to these two-digit numbers, decomposing them into tens and ones.

- **Teaching point 1:** There is a set counting sequence for counting to 100 and beyond.
- **Teaching point 2:** Objects can be counted efficiently by making groups of ten. The digits in the numbers 20–99 tell us about their value.
- **Teaching point 3:** Each number on the 0–100 number line has a unique position.
- **Teaching point 4:** The relative size of two two-digit numbers can be determined by first examining the tens digits and then, if necessary, examining the ones digits, with reference to the cardinal or ordinal value of the numbers.
- **Teaching point 5:** Each two-digit number can be partitioned into a tens part and a ones part.
- **Teaching point 6:** The tens and ones structure of two-digit numbers can be used to support additive calculation.

1.10 Composition of numbers: 11–19

Explore the ten-and-a-bit nature of the numbers 11–19, using the partitioning structure; apply number facts within ten to addition and subtraction of single-digit numbers to/from the numbers 11–19.

- **Teaching point 1:** The digits in the numbers 11–19 tell us about their value.
- **Teaching point 2:** The numbers 11–19 can be formed by combining a ten and ones, and can be partitioned into a ten and ones.
- **Teaching point 3:** A number is even if the ones digit is even; it *can* be made from groups of two. A number is odd if the ones digit is odd; it *can't* be made from groups of two.
- **Teaching point 4:** Doubling the numbers 6–9 (inclusive) gives an even teen number; halving an even teen number gives a number from six to nine (inclusive).
- **Teaching point 5:** Addition and subtraction facts within 10 can be applied to addition and subtraction within 20.

Year 2

1.11 Addition and subtraction: bridging 10

Apply the aggregation and augmentation structures of addition to three single-digit numbers, exploring commutativity and associativity, to work towards strategies for adding and subtracting across ten.

- **Teaching point 1:** Addition of three addends can be described by an aggregation story with three parts.
- **Teaching point 2:** Addition of three addends can be described by an augmentation story with a *'first…, then…, then…, now…'* structure.
- **Teaching point 3:** The order in which addends (parts) are added or grouped does not change the sum (associative and commutative laws).
- **Teaching point 4:** When we are adding three numbers, we choose the most efficient order in which to add them, including identifying two addends that make ten (combining).
- **Teaching point 5:** We can add two numbers which bridge the tens boundary by using a 'make ten' strategy.
- **Teaching point 6:** We can subtract across the tens boundary by subtracting *through* ten or subtracting *from* ten.

1.12 Subtraction as difference

Introduce children to subtraction as difference, the third and final subtraction structure; review consecutive numbers, as well as consecutive odd/even numbers, in the context of difference.

- **Teaching point 1:** Difference compares the number of objects in one set with the number of objects in another set; or the difference between two measures.
- **Teaching point 2:** Difference is one of the structures of subtraction.
- **Teaching point 3:** Consecutive whole numbers have a difference of one; consecutive odd/even numbers have a difference of two.
- **Teaching point 4:** We can apply the structure of difference to compare data.

1.13 Addition and subtraction: two-digit and single-digit numbers

Build on segments 1.8, 1.9 and 1.10 to equip children with useful strategies for addition and subtraction of a single-digit number to/from two-digit numbers.

- **Teaching point 1:** Knowledge of the number line, and quantity values of numbers, can be applied to add/subtract one to/from a given two-digit number.
- **Teaching point 2:** Known facts for the numbers *within* ten can be applied to addition/subtraction of a single-digit number to/from a two-digit number.
- **Teaching point 3:** Knowledge of numbers which sum to ten can be applied to the addition of a single-digit number and two-digit number that sum to a multiple of ten, or subtraction of a single-digit number from a multiple of ten.
- **Teaching point 4:** Known strategies for addition or subtraction bridging ten can be applied to addition or subtraction bridging a multiple of ten.

1.14 Addition and subtraction: two-digit numbers and multiples of ten

Explore counting on, and back, in ten from any two-digit number; apply number facts within ten to the addition and subtraction of multiples of ten.

- **Teaching point 1** When finding ten more or ten less than any two-digit number, the ones digit does not change.
- **Teaching point 2:** When ten is added or subtracted to/from a two-digit number, the tens digit changes and the ones digit stays the same.
- **Teaching point 3:** Knowledge of number facts within ten can be applied to adding or subtracting multiples of ten to/from a two-digit number.
- **Teaching point 4:** Two-digit numbers can be partitioned in different ways.

1.15 Addition: two-digit and two-digit numbers

Build on segments 1.13 and 1.14 to equip children with useful strategies for addition of two or more two-digit numbers, partitioning two-digit numbers into tens and ones before calculation.

- **Teaching point 1:** Known strategies can be combined to add two multiples of ten to two single-digit numbers.
- **Teaching point 2:** Two two-digit numbers can be added by partitioning one or both of them into tens and ones.

1.16 Subtraction: two-digit and two-digit numbers

Build on segments 1.13 and 1.14 to equip children with useful strategies for subtraction of one two-digit number from another, partitioning two-digit numbers into tens and ones before calculation.

- **Teaching point 1:** Known strategies can be used to subtract a multiple of ten and a single-digit number from a two-digit number.
- **Teaching point 2:** A two-digit number can be subtracted from a two-digit number by partitioning the subtrahend into tens and ones.

Year 3

1.17 Composition and calculation: 100 and bridging 100

Explore the additive and multiplicative composition of 100; draw on known strategies and number facts to calculate across the 100 boundary.

- **Teaching point 1:** There are ten tens in 100; there are 100 ones in 100. 100 can also be composed multiplicatively from 50, 25 or 20, units that are commonly used in graphing and measures.
- **Teaching point 2:** Known addition facts can be used to calculate complements to 100.
- **Teaching point 3:** Known strategies for addition and subtraction across the tens boundary can be combined with unitising to count and calculate across the hundreds boundary in multiples of ten.
- **Teaching point 4:** Knowledge of two-digit numbers can be extended to count and calculate across the hundreds boundary from/to any two-digit number in ones or tens.

1.18 Composition and calculation: three-digit numbers

Explore the composition of three-digit numbers; use place-value and partitioning knowledge to support additive calculation, and extend known additive strategies to three-digit numbers.

- **Teaching point 1:** Three-digit numbers can be composed additively from hundreds, tens and ones; this structure can be used to support additive calculation.
- **Teaching point 2:** Each number on the 0 to 1,000 number line has a unique position.
- **Teaching point 3:** The smallest three-digit number is 100, and the largest three-digit number is 999; the relative size of two three-digit numbers can be determined by examining the hundreds digits, then the tens digits, and then the ones digits, as necessary.
- **Teaching point 4:** Three-digit multiples of ten can be expressed multiplicatively and additively, in terms of tens or hundreds.
- **Teaching point 5:** Known facts and strategies for addition and subtraction within and across ten, and within and across 100, can be used to support additive calculation within 1,000.
- **Teaching point 6:** Familiar counting sequences can be extended up to 1,000.

1.19 Securing mental strategies: calculation up to 999

Build on segments 1.15 and 1.16 to equip children with useful calculation strategies for bridging hundreds boundaries, and three-digit numbers; continue to use the partitioning structure to facilitate calculation.

- **Teaching point 1:** Known partitioning strategies for adding two-digit numbers within 100 can be extended to the mental addition of two-digit numbers that bridge 100, and addition of three-digit numbers.
- **Teaching point 2:** Transforming addition calculations into equivalent calculations can support efficient mental strategies.
- **Teaching point 3:** Subtraction calculations can be solved using a ‘finding the difference’ strategy; this can be thought of as ‘adding on’ to find a missing part.
- **Teaching point 4:** The order of addition and subtraction steps in a multi-step calculation can be chosen or manipulated such as to simplify the arithmetic.

1.20 Algorithms: column addition

Introduce children to the column algorithm for addition calculations, applying the algorithm to a variety of aggregation and augmentation contexts for two-digit and three-digit numbers; explore regrouping (column total is ten or greater) in detail.

- **Teaching point 1:** Any numbers can be added together using an algorithm called *'column addition'*.
- **Teaching point 2:** The digits of the addends must be aligned correctly before the algorithm is applied.
- **Teaching point 3:** In column addition, the digits of the addends are added working from the least significant digit (on the right) to the most significant digit (on the left).
- **Teaching point 4:** If any column sums to ten or greater, we must *'regroup'*.
- **Teaching point 5:** The numbers within each column should be added in the most efficient order.

1.21 Algorithms: column subtraction

Introduce children to the column algorithm for subtraction calculations, applying the algorithm to a variety of partitioning, reduction and difference contexts for two-digit and three-digit numbers; explore exchange (insufficient quantity to subtract from in a column) in detail.

- **Teaching point 1:** One number can be subtracted from another using an algorithm called *'column subtraction'*; the digits of the minuend and subtrahend must be aligned correctly; the algorithm is applied working from the least significant digit (on the right) to the most significant digit (on the left).
- **Teaching point 2:** If there is an insufficient number of any unit to subtract from in a given column, we must exchange from the column to the left.

Year 4

1.22 Composition and calculation: 1,000 and four-digit numbers

Explore the composition of 1,000 and four-digit numbers, using the partitioning structure, and make links to measures; introduce children to calculation across thousands boundaries, and extend column algorithms and rounding to four-digit numbers.

- **Teaching point 1:** Ten hundreds make 1,000, which can also be decomposed into 100 tens and 1,000 ones.
- **Teaching point 2:** When multiples of 100 are added or subtracted, the sum or difference is always a multiple of 100.
- **Teaching point 3:** Numbers over 1,000 have a structure that relates to their size. This means they can be ordered, composed and decomposed.
- **Teaching point 4:** Numbers can be rounded to simplify calculations or to indicate approximate sizes.
- **Teaching point 5:** Calculation approaches learnt for three-digit numbers can be applied to four-digit numbers.
- **Teaching point 6:** 1,000 can also be composed multiplicatively from 500s, 250s or 200s, units that are commonly used in graphing and measures.

1.23 Composition and calculation: tenths

Introduce children to tenths using both the partitioning structure and ideas of place value; apply additive facts and strategies, including column algorithms, and rounding to numbers with tenths.

- **Teaching point 1:** When one is divided into ten equal parts, each part is one tenth of the whole.
- **Teaching point 2:** Tenths can be expressed as decimal fractions; the number written '0.1' is one tenth; one is ten times the size of 0.1.
- **Teaching point 3:** We can count in tenths up to and beyond one.
- **Teaching point 4:** Numbers with tenths can be composed additively and multiplicatively.
- **Teaching point 5:** Known facts and strategies, including column algorithms, can be applied to calculations for numbers with tenths.
- **Teaching point 6:** Numbers with tenths can be rounded to the nearest whole number by examining the value of the tenths digit.

1.24 Composition and calculation: hundredths and thousandths

Building on segment 1.23, introduce children to hundredths (and thousandths) using both the partitioning structure and ideas of place value; apply additive facts and strategies, including column algorithms, and rounding to numbers with hundredths (and thousandths).

- **Teaching point 1:** When one is divided into 100 equal parts, each part is one hundredth of the whole. When one tenth of a whole is divided into ten equal parts, each part is one hundredth of the whole.
- **Teaching point 2:** Hundredths can be expressed as decimal fractions; the number written '0.01' is one hundredth; one is one hundred times the size of 0.01; 0.1 is ten times the size of 0.01.
- **Teaching point 3:** We can count in hundredths up to and beyond one.
- **Teaching point 4:** Numbers with hundredths can be composed additively and multiplicatively.

- **Teaching point 5:** Numbers with tenths and hundredths are commonly used in measurement, scales and graphing contexts.
- **Teaching point 6:** Known facts and strategies, including column algorithms, can be applied to calculations for numbers with hundredths; the same approaches can be used for numbers with hundredths as are used for numbers with tenths.
- **Teaching point 7:** Numbers with hundredths can be rounded to the nearest tenth by examining the value of the hundredths digit or to the nearest whole number by examining the value of the tenths digit.
- **Teaching point 8:** When one is divided into 1,000 equal parts, each part is one thousandth of the whole. Knowledge and strategies for numbers with tenths and hundredths can be applied to numbers with thousandths.

1.25 Addition and subtraction: money

Building on segments 1.23 and 1.24, introduce children to conventions for expressing monetary value and explore the equivalence of 100 p and £1; encourage children to select column algorithms or equivalent calculations where most appropriate.

- **Teaching point 1:** One penny is one hundredth of a pound; conventions for expressing quantities of money are based on expressing numbers with tenths and hundredths.
- **Teaching point 2:** Equivalent calculation strategies for addition can be used to efficiently add commonly-used prices.
- **Teaching point 3:** The ‘working forwards’/‘finding the difference’ strategy for subtraction is an efficient way to calculate the change due when paying in whole pounds or notes.
- **Teaching point 4:** Column methods can be used to add and subtract quantities of money.
- **Teaching point 5:** Finding change when purchasing several items uses the part–part–(part–) whole structure.

Year 5

1.26 Composition and calculation: multiples of 1,000 up to 1,000,000

Explore the composition of six-digit, whole-thousand numbers, using the partitioning structure; apply knowledge and strategies from segments 1.17 and 1.18 combined with unitising in 1,000s, as well as column methods and rounding.

- **Teaching point 1:** Understanding of numbers composed of hundred thousands, ten thousands and one thousands can be supported by making links to numbers composed of hundreds, tens and ones.
- **Teaching point 2:** Multiples of 1,000 up to 1,000,000 can be placed in the linear number system by drawing on knowledge of the place of numbers up to 1,000 in the linear number system.
- **Teaching point 3:** Numbers can be ordered and compared using knowledge of their composition and of their place in the linear number system.
- **Teaching point 4:** Calculation approaches for numbers up to 1,000 can be applied to multiples of 1,000 up to 1,000,000.
- **Teaching point 5:** Numbers can be rounded to simplify calculations or to indicate approximate sizes.
- **Teaching point 6:** Known patterns can be used to divide 10,000 and 100,000 into two, four and five equal parts. These units are commonly used in graphing and measures.

1.27 Negative numbers: counting, comparing and calculating

Introduce children to negative numbers, making links to everyday contexts; explore addition and subtraction below zero and across zero.

- **Teaching point 1:** Positive and negative numbers can be used to represent change.
- **Teaching point 2:** Our number system includes numbers that are less than zero; these are negative numbers. Numbers greater than zero are positive numbers.
- **Teaching point 3:** The negative/minus symbol (–) is placed before a numeral to indicate that the value is a negative number.
- **Teaching point 4:** Negative numbers can be shown on horizontal scales; numbers to the left of zero are negative (less than zero) and numbers to the right of zero are positive (greater than zero). The larger the value of the numeral after the negative/minus symbol, the further the number is from zero.
- **Teaching point 5:** Knowledge of the positions of positive and negative numbers in the number system can be used to calculate intervals across zero.
- **Teaching point 6:** Negative numbers are used in coordinate and graphing contexts.

1.28 Common structures and the part–part–whole relationship

Extend the part–part–part–whole structure (three or more parts) to solve missing part/whole problems in a range of contexts; draw on number composition and additive concepts from across the spine, focusing on the structural equivalence of the problems.

- **Teaching point 1:** Mathematical relationships encountered at primary level are either additive or multiplicative; both of these can be observed within the structure of part–part–whole relationships.
- **Teaching point 2:** Problems in many different contexts can be solved by adding together the parts to find the whole. Different strategies can be used to calculate the whole, but the structure of the problem remains the same.
- **Teaching point 3:** If the value of the whole is known, along with the values of all but one of the parts, the value of the missing part can be calculated. Different strategies can be used to calculate the missing part, but the structure of the problem remains the same.
- **Teaching point 4:** Problems in many different contexts have the 'missing-part' structure.

1.29 Using equivalence and the compensation property to calculate

Explore the effect on the sum of changing the value of one or both addends; explore the effect on the difference of changing the value of the minuend, the subtrahend or both. Apply knowledge of compensation properties and inverse operations to calculate and balance equations.

- **Teaching point 1:** If one addend is increased and the other is decreased by the same amount, the sum stays the same. (same sum)
- **Teaching point 2:** If one addend is increased (or decreased) and the other is kept the same, the sum increases (or decreases) by the same amount.
- **Teaching point 3:** If the minuend and subtrahend are changed by the same amount, the difference stays the same. (same difference)
- **Teaching point 4:** If the minuend is increased (or decreased) and the subtrahend is kept the same, the difference increases (or decreases) by the same amount.
- **Teaching point 5:** If the minuend is kept the same and the subtrahend is increased (or decreased), the difference decreases (or increases) by the same amount.
- **Teaching point 6:** The value of the expressions on each side of an equals symbol must be the same; addition and subtraction are inverse operations. We can use this knowledge to balance equations and solve problems.

Year 6

1.30 Composition and calculation: numbers up to 10,000,000

Building on segment 1.26, explore six-digit numbers that are not whole thousands, and then extend to seven-digit numbers; apply additive facts and strategies, including column algorithms, and rounding to these numbers.

- **Teaching point 1:** Patterns seen in other powers of ten can be extended to the unit 1,000,000.
- **Teaching point 2:** Seven-digit numbers can be written, read and ordered by identifying the number of millions, the number of thousands and the number of hundreds, tens and ones.
- **Teaching point 3:** The digits in a number indicate its structure so it can be composed and decomposed.
- **Teaching point 4:** Knowledge of crossing thousands boundaries can be used to work to and across millions boundaries.
- **Teaching point 5:** Sometimes numbers are rounded as approximations to eliminate an unnecessary level of detail; rounded numbers are also used to give an estimate or average. At other times, precise readings are useful.
- **Teaching point 6:** Fluent calculation requires the flexibility to move between mental and written methods according to the specific numbers in a calculation.

1.31 Problems with two unknowns

Equip children with strategies for solving problems with two unknowns, including using the bar model to represent relationships between known numbers, and working systematically.

- **Teaching point 1:** Problems with two unknowns can have one solution or more than one solution (or no solution). A relationship between the two unknowns can be described in different ways, including additively and multiplicatively.
- **Teaching point 2:** Model drawing can be used to expose the structure of problems with two unknowns.
- **Teaching point 3:** A problem with two unknowns has only one solution if the sum of the two unknowns and the difference between them is given (*'sum-and-difference problems'*) or if the sum of the two unknowns and a multiplicative relationship between them is given (*'sum-and-multiple problems'*).
- **Teaching point 4:** Other problems with two unknowns have only one solution.
- **Teaching point 5:** Some problems with two unknowns can't easily be solved using model drawing but can be solved by a 'trial-and-improvement' approach; these problems may have one solution, several solutions or an infinite number of solutions.